Modeling Multipartite Entanglement in Quantum Protocols using Evolving Entangled Hypergraphs

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November 8, 2017 - Verona, Italy

Quantum Techniques in Machine Learning

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Motivation

Motivation

Multipartite Entanglement $\neq$ Bipartite Entanglement

Motivation

- Graph structures are widely used in many research fields $\rightarrow$ fast and scalable for retrieving and analyzing recurring patterns.
- In this work we try to provide a notion of multipartite entanglement classification using hypergraphs.
- In the second part we will use this classification as a starting point to provide a more complex structure suitable to model the dynamic of (open) quantum systems in which multipartite entanglement is an emergent behavior.
- We finally investigate some future application in quantum cryptography and quantum model checking.

Introduction

Entanglement Measures

- Concurrence: for a general 2-qubit state

$$\mathcal{C} = |\langle\Psi|\sigma_y \otimes \sigma_y|\Psi^*\rangle|$$

- Tangle: for a 3-qubit state

$$\tau = \mathcal{C}^2_{A(BC)} - \mathcal{C}^2_{AB} - \mathcal{C}^2_{AC}$$

- Global entanglement: for a N-qubit pure state partitioned into two blocks S and $\bar{S}$ with m and $N - m$ qubits respectively.

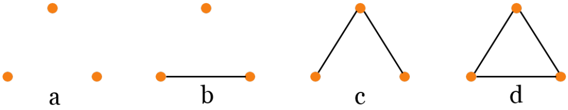
degree of entanglement of S to the rest: $$\eta_{S\bar{S}} = \frac{2^m}{2^m - 1}\left(1 - Tr(\varrho_S^2)\right)$$

W.K. Wootters, PRL 80, 2245 (1998)
V. Coffman, J. Kundu, W.K. Wootters, PRA 61, 052306 (2000)
P.J. Love et al., QIP 6, 187 (2007)

Entangled Graphs

Goal: to write a pure state for every possible graph where:

- Vertex $\longleftrightarrow$ Qubit
- Edge $\longleftrightarrow$ Bipartite entanglement



$$\begin{cases} a) \left\{ \begin{array}{l} |\text{Sep}\rangle = |\varphi_1\rangle \otimes |\varphi_2\rangle \otimes |\varphi_3\rangle \\ |\text{GHZ}\rangle = \alpha|000\rangle + \beta|111\rangle \end{array} \right\} \text{ambiguity!} \\ b) \quad |\text{BS}\rangle = |\text{Bell State}\rangle \otimes |\varphi\rangle \\ c) \quad |\text{Star}\rangle = \alpha|000\rangle + \beta|100\rangle + \gamma|110\rangle + \delta|111\rangle \\ d) \quad |\text{W}\rangle = \alpha|001\rangle + \beta|010\rangle + \gamma|100\rangle \end{cases}$$

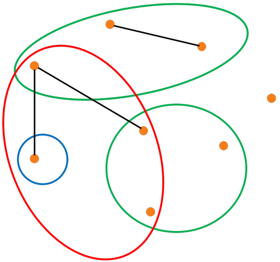
Weighted entangled graphs: Edges are weighted by concurrence

M. Plesch & V. Bužek, PRA 67, 012322 (2003)

Map: States $\longleftrightarrow$ Hypergraphs

Hypergraph $\rightarrow$ **generalization** of a graph $H = (V, E)$ where:

- V is a set of elements called vertices
- E is a subset of $\mathcal{P}(V)$ called hyperedges ($\mathcal{P}$ is the power set of V)

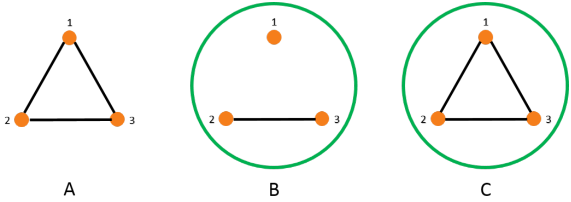


Connected hypergraph: existence of a path between every pair of vertices

Entangled Hypergraphs

Entangled hypergraph (EH) is a generalization of entangled graph where:

- Vertex $\longleftrightarrow$ Qubit
- Hyperedge $\longleftrightarrow$ Multipartite entanglement



$$
\begin{cases}
|A\rangle = \alpha|000\rangle + \beta(|101\rangle + |110\rangle) \\
|B\rangle = \alpha|000\rangle + \beta|001\rangle + \gamma(|110\rangle + |111\rangle) \\
|C\rangle = \alpha|000\rangle + \beta(|011\rangle + |101\rangle + |110\rangle)
\end{cases}
$$

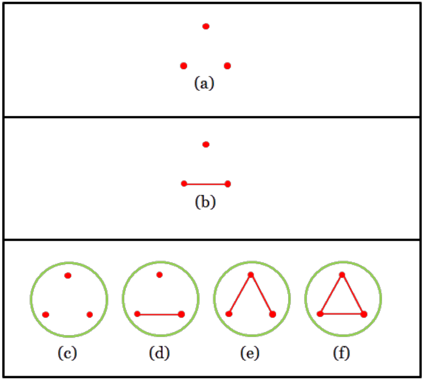
Classification of 3-qubit entanglement

The generalized Schmidt decomposition for 3-qubit pure state is as follow

$$|\Psi\rangle_3 = \lambda_0|000\rangle + \lambda_1 e^{i\phi}|100\rangle + \lambda_2|101\rangle + \lambda_3|110\rangle + \lambda_4|111\rangle$$
$$\lambda_i \geq 0, \quad 0 \leq \phi \leq \pi, \quad \sum \lambda_i^2 = 1$$

$\mathcal{C}_{12}$	$\mathcal{C}_{13}$	$\mathcal{C}_{23}$
(λ_0 , λ_3)	(λ_0 , λ_2)	(λ_1 , λ_4)
		(λ_2 , λ_3)

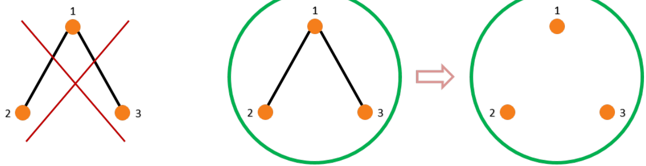
$$\begin{cases} |A\rangle = |\varphi_1\rangle \otimes |\varphi_2\rangle \otimes |\varphi_3\rangle \\ |B\rangle = (\lambda_0|00\rangle + \lambda_3|11\rangle) \otimes |0\rangle \\ |C\rangle = \lambda_0|000\rangle + \lambda_4|111\rangle \\ |D\rangle = \lambda_0|000\rangle + \lambda_3|110\rangle + \lambda_4|111\rangle \\ |E\rangle = \lambda_0|000\rangle + \lambda_1|100\rangle + \lambda_3|110\rangle \\ \qquad +\lambda_4|111\rangle \\ |F\rangle = \lambda_0|000\rangle + \lambda_2|101\rangle + \lambda_3|110\rangle \end{cases}$$



M. Gharahi Ghahi & S.J. Akhtarshenas, EPJD 70, 54 (2016)

Forbidden Entangled Hypergraphs

There is no corresponding pure state to star shape graph:



$$|\psi\rangle = \cos^2\theta|000\rangle + i\,\sin\theta\cos\theta(|011\rangle + |101\rangle) - \sin^2\theta|110\rangle$$

$$\begin{cases} \mathcal{C}_{12} = 2\,|\sin^3\theta\cos\theta - \sin\theta\cos^3\theta| \\ \mathcal{C}_{13} = 2\,|\sin^3\theta\cos\theta - \sin\theta\cos^3\theta| \\ \mathcal{C}_{12} = 0 \\ \tau = 16\,\sin^4\theta\cos^4\theta \end{cases} \quad \text{if } \theta = \frac{\pi}{4} \Rightarrow \quad \begin{cases} \mathcal{C}_{12} = 0 \\ \mathcal{C}_{13} = 0 \\ \mathcal{C}_{12} = 0 \\ \tau = 1 \end{cases}$$

Evolving Entangled Hypergraphs

Motivation & Definition

Evolving Entangled Hypergraphs (EEH) are inspired by

- Entangled Hypergraphs
- Hypergraph States

We take into account not only correlations (entanglement) but also interactions (evolution) of the qubits.

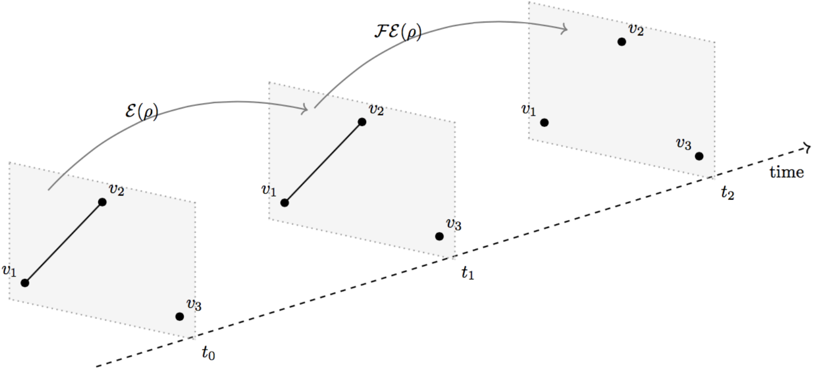
EEHs allow us to verify if the entanglement properties are preserved by the time evolution.

An EEH is a causal multilayer Hypergraph in which

- each layer L_i represents the EH at time t_i
- the hyperedges from a layer to the following one are labeled with the CPTP[1] linear map acting on the states of EH.

[1] : completely positive and trace preserving

Example



Where ρ refers to a given state of three particles and the transformations $\mathcal{E}$ and $\mathcal{F}$ are CPTP maps.

Advantages

- EHs are a finitary method to classify entanglement
- It is **not** required to compute the mathematical representation of the operators
- EEH can be built algorithmically $\rightarrow$ we are planning to build a tool do automate the process
- EEH can be used in model checking of quantum protocols and quantum dynamics
- EEH can be further investigated to deal with system of identical particles

Perspective works - Model Checking with EEH

We suppose that model checking of Quantum protocols can be perfomed using both EEH and Spatial-Temporal logics (e.g., SSTL):

- spatial locations of qubits are represented by the vertices of the EH (which can be interpreted as qubit positions)
- the evolution modeled by EEH describes the temporal behavior of the system
- the property entanglement can be considered an emergent behavior of the system (e.g., pattern formation)

Perspective works - Application

- Quantum Cryptography - reliability and robustness of QKD protocols
- Topological (possibly multipartite) entanglement classification
- Assessment of entanglement property in system of identical particles

Thanks!